



FIVE RULES FOR THE EVOLUTION OF COOPERATION

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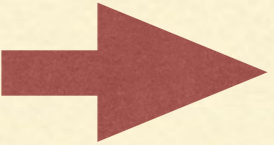
OUTLINE

- Natural selection
 - Kin selection
 - Direct reciprocity
 - Indirect reciprocity
 - Network reciprocity
 - Group selection
 - Mathematical framework
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NATURAL SELECTION

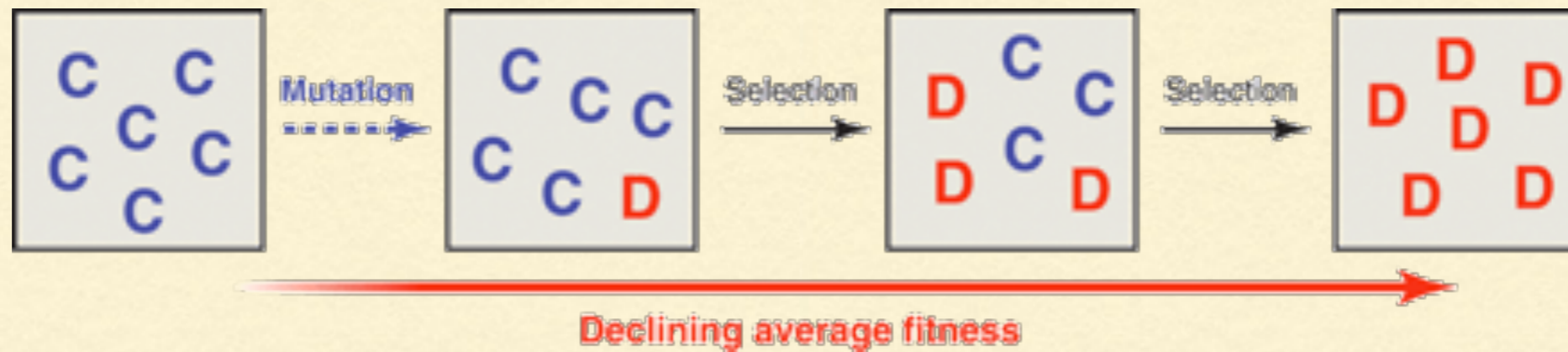
- A cooperator is someone who pays a cost, c , for another individual to receive a benefit, b .
 - A defector has no cost and does not deal out benefits.
 - Cost and benefit are measured in terms of fitness.
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PRISONERS' DILEMMA



	C	D
C	$b-c$	$-c$
D	b	0

$b > c > 0$



- i cooperators and $N - i$ defectors
 - $f_C = [b(i - 1)/(N - 1)] - c$
 - $f_D = bi/(N - 1)$
-

KIN SELECTION

“I will jump into the river to save
two brothers or eight cousins.”

–*J.B.S. Haldane, 1930s*

- the coefficient of relatedness, r , must exceed the cost-to-benefit ratio of the altruistic act:

$$r > c/b$$

DIRECT RECIPROcity

- It is unsatisfactory to have a theory that can only explain cooperation among relatives.
 - repeated Prisoner's Dilemma (repeated encounters between the same two individuals)
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- tit-for-tat
 - generous-tit-for-tat (cooperates although you have defected with probability $1 - c/b$)
 - win-stay, lose-shift
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- Direct reciprocity can only lead to the evolution of cooperation if the probability, w , of another encounter between the same two individuals exceeds the cost-to-benefit ratio of the altruistic act:

$$w > c/b$$

INDIRECT RECIPROcity

- Direct reciprocity relies on repeated encounters between the same two individuals, and both individuals must be able to provide help.
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- Randomly chosen, pairwise encounters, where the same two individuals need not meet again.
- One individual acts as donor the other as recipient. The donor can decide whether or not to cooperate.
- Indirect reciprocity can only promote cooperation if the probability, q , to know someone's reputation exceeds the cost-to-benefit ratio of the altruistic act:

$$q > c/b$$

NETWORK RECIPROCALITY

- A cooperator pays a cost, c , for each neighbor to receive a benefit, b .
- Defectors have no costs, and their neighbors receive no benefits.
- The benefit-to-cost ratio must exceed the average number of neighbors, k , per individual:

$$b/c > k$$

GROUP SELECTION

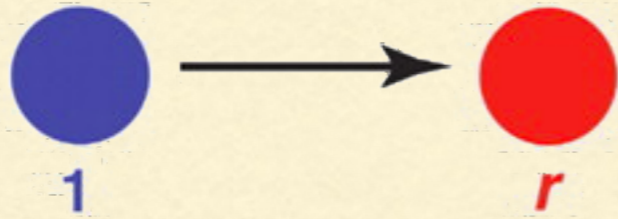
- Cooperators help others in their own group. Defectors do not help.
 - Individuals reproduce proportional to their payoff.
 - Offspring are added to the same group.
 - If a group reaches a certain size it can split into two.
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- only individuals reproduce, but selection emerges on two levels.
- In particular, pure cooperator groups grow faster than pure defector groups, while in any mixed group defectors reproduce faster than cooperators.
- if n is the maximum group size and m the number of groups, then group selection allows evolution of cooperation provided

$$b/c > 1+n/m$$

Smaller group sizes and larger numbers of groups favor cooperators

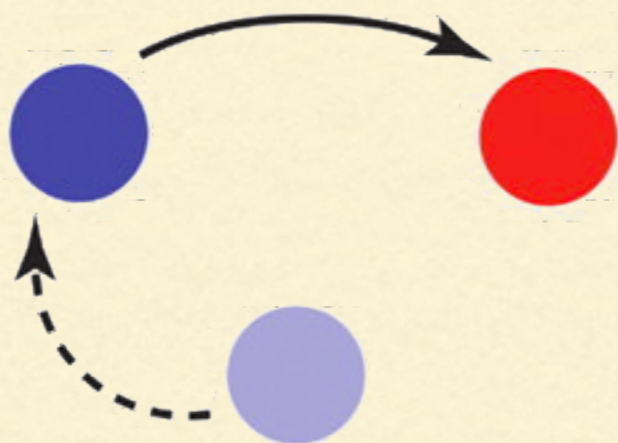
Kin selection



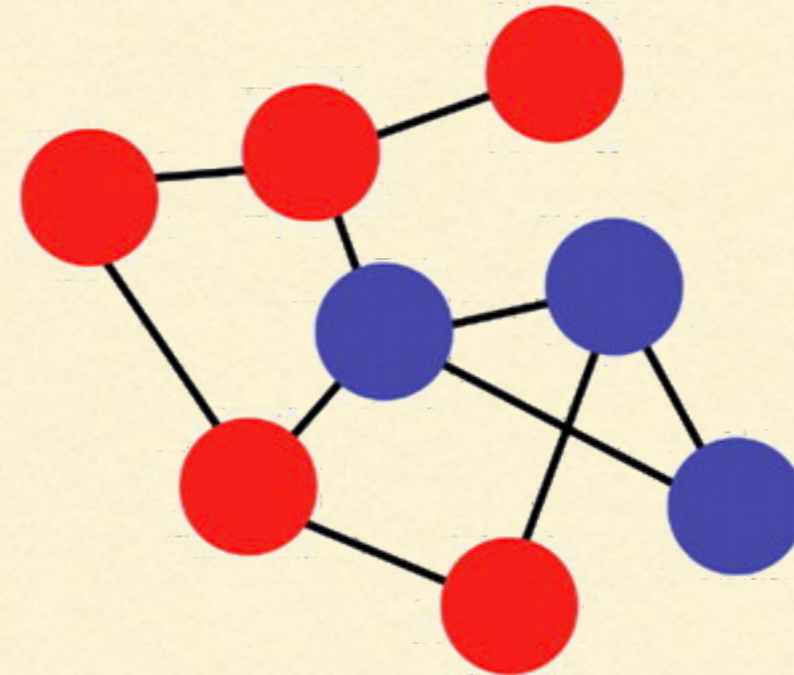
Direct reciprocity



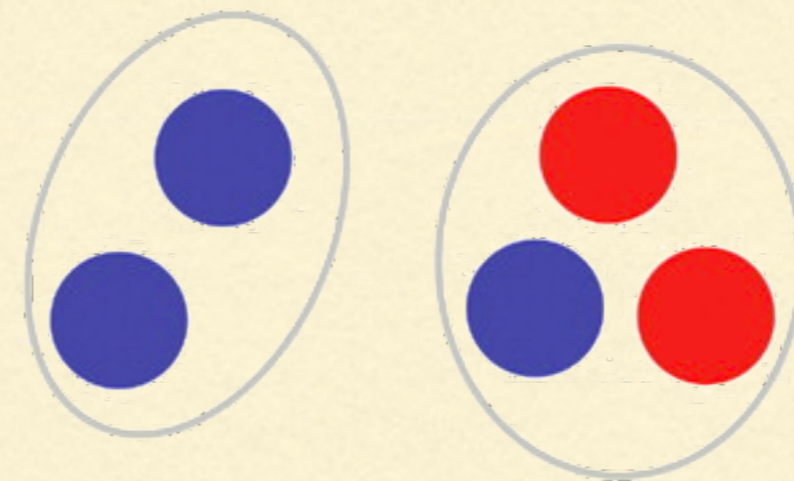
Indirect reciprocity



Network reciprocity



Group selection



● Cooperators

● Defectors

MATHEMATICAL FRAMEWORK

$$\begin{array}{c}
 C \\
 D
 \end{array}
 \begin{array}{cc}
 C & D \\
 \left(\begin{array}{cc}
 \alpha & \beta \\
 \gamma & \delta
 \end{array} \right)
 \end{array}$$

$$\alpha > \gamma$$

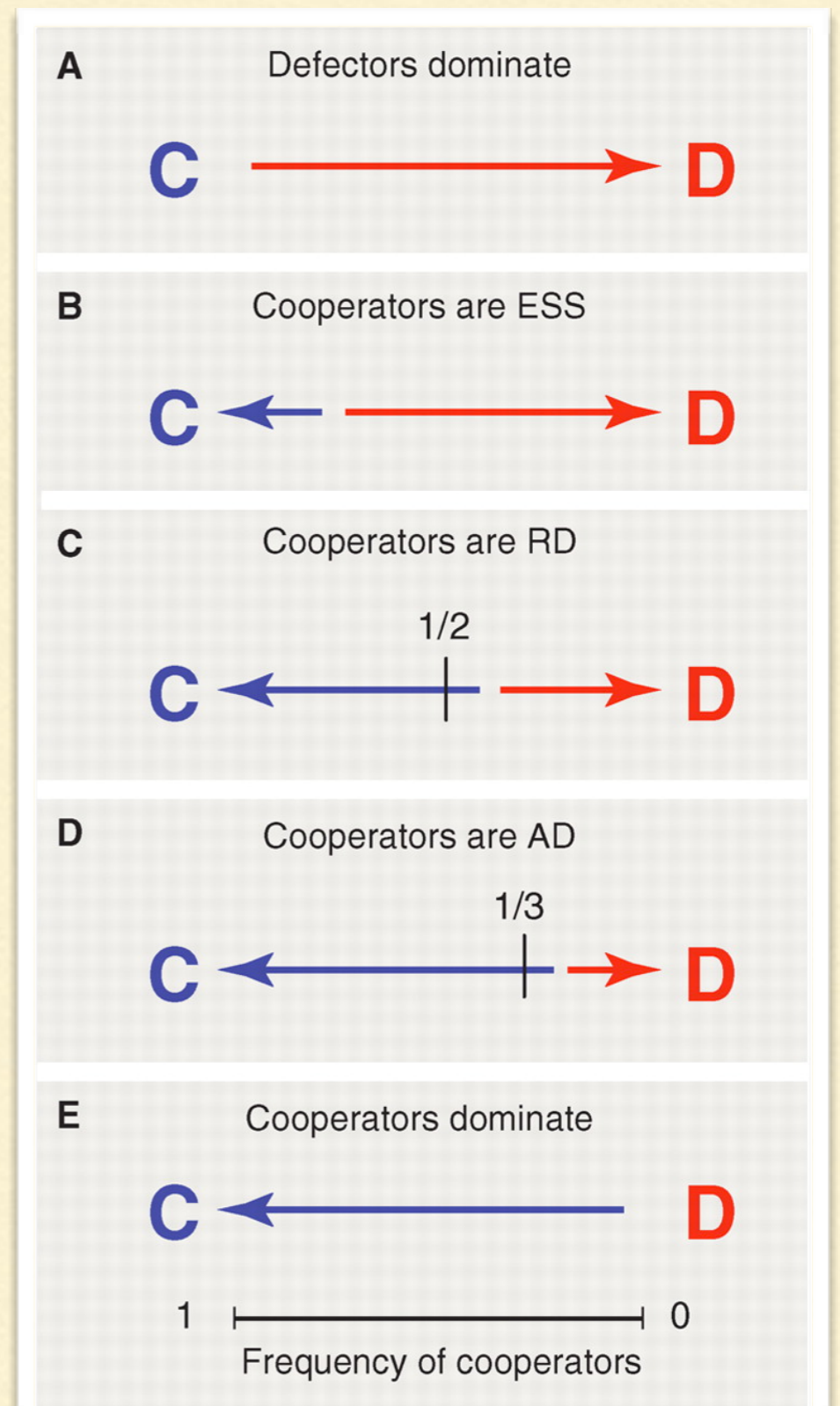
evolutionarily stable strategy (ESS)

$$\alpha + \beta > \gamma + \delta$$

risk-dominant (RD)

$$\alpha + 2\beta > \gamma + 2\delta$$

advantageous (AD)



Payoff matrix

Cooperation is ...

ESS

RD

AD

Kin selection

	<i>C</i>	<i>D</i>
<i>C</i>	$(b - c)(1 + r)$	$br - c$
<i>D</i>	$b - rc$	0

$$\frac{b}{c} > \frac{1}{r}$$

$$\frac{b}{c} > \frac{1}{r}$$

$$\frac{b}{c} > \frac{1}{r}$$

r...genetic relatedness

Direct reciprocity

	<i>C</i>	<i>D</i>
<i>C</i>	$(b - c) / (1 - w)$	$-c$
<i>D</i>	b	0

$$\frac{b}{c} > \frac{1}{w}$$

$$\frac{b}{c} > \frac{2 - w}{w}$$

$$\frac{b}{c} > \frac{3 - 2w}{w}$$

w...probability of next round

Indirect reciprocity

	<i>C</i>	<i>D</i>
<i>C</i>	$b - c$	$-c(1 - q)$
<i>D</i>	$b(1 - q)$	0

$$\frac{b}{c} > \frac{1}{q}$$

$$\frac{b}{c} > \frac{2 - q}{q}$$

$$\frac{b}{c} > \frac{3 - 2q}{q}$$

q...social acquaintanceship

Network reciprocity

	<i>C</i>	<i>D</i>
<i>C</i>	$b - c$	$H - c$
<i>D</i>	$b - H$	0

$$\frac{b}{c} > k$$

$$\frac{b}{c} > k$$

$$\frac{b}{c} > k$$

k...number of neighbors

$$H = \frac{(b - c)k - 2c}{(k + 1)(k - 2)}$$

Group selection

	<i>C</i>	<i>D</i>
<i>C</i>	$(b - c)(m + n)$	$(b - c)m - cn$
<i>D</i>	bn	0

$$\frac{b}{c} > 1 + \frac{n}{m}$$

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$$\frac{b}{c} > 1 + \frac{n}{m}$$

n...group size
m...number of groups

THANKS
