



Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent

William H Press and Freeman J Dyson.
Proceedings of the National Academy of Sciences of the United States of
America, 2012, 109(26): 10409–10413.

Presented by Wang Jiasheng

Agenda

- 上次留下的几个问题
- Press 和 Dyson 的文章
 - 预备知识
 - Zero-Determinant Strategies
 - 如何作用
 - 总结和讨论

- A cooperator pays a cost, c , for each neighbor to receive a benefit, b .
- Defectors have no costs, and their neighbors receive no benefits.
- The benefit-to-cost ratio must exceed the average number of neighbors, k , per individual:

$$\begin{array}{c} C \\ D \end{array} \begin{array}{cc} C & D \\ \left(\begin{array}{cc} b-c & -c \\ b & 0 \end{array} \right) \end{array}$$

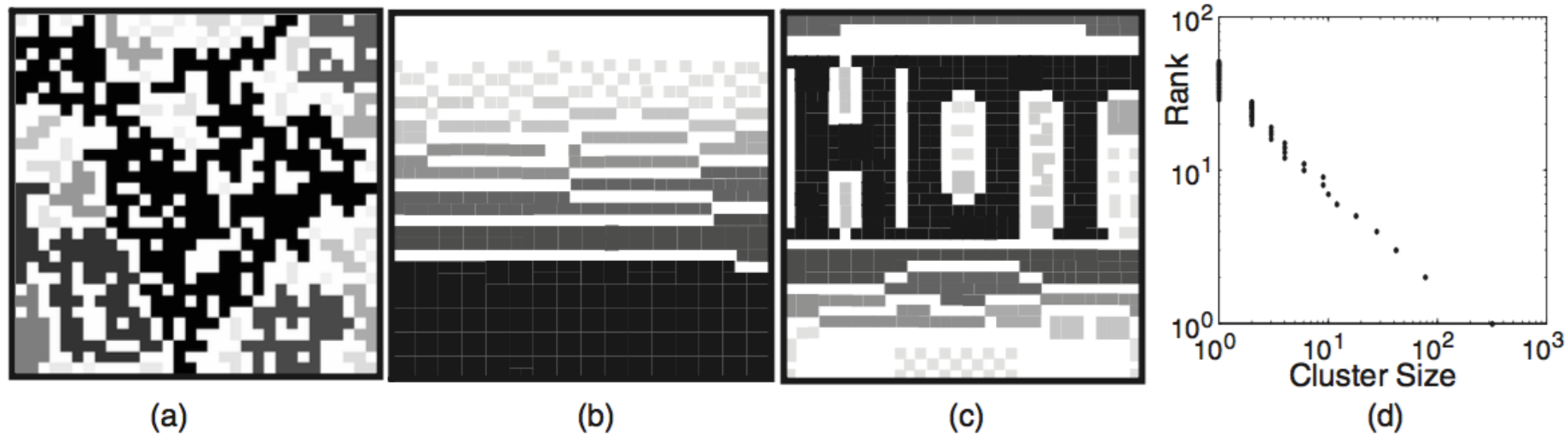
$$b/c > k$$



$$b > kc$$

无标度 (Scale-free)

- 临界相变 (critical phase transitions)、逾渗 (percolation)、分形 (fractal)



Iterated Prisoner's Dilemma (IPD)

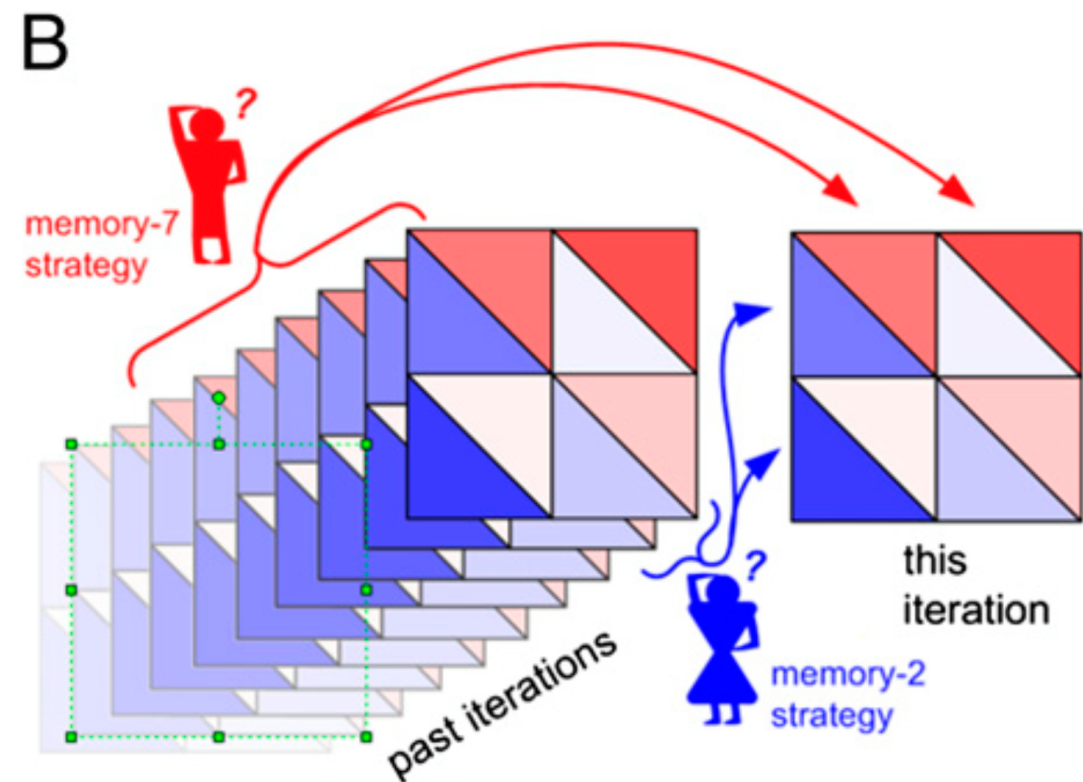
- (T, R, P, S)
- $T > R > P > S$
- $2R > T + S$

A

	c	d
c	3 (R)	5 (T)
d	0 (S)	1 (P)

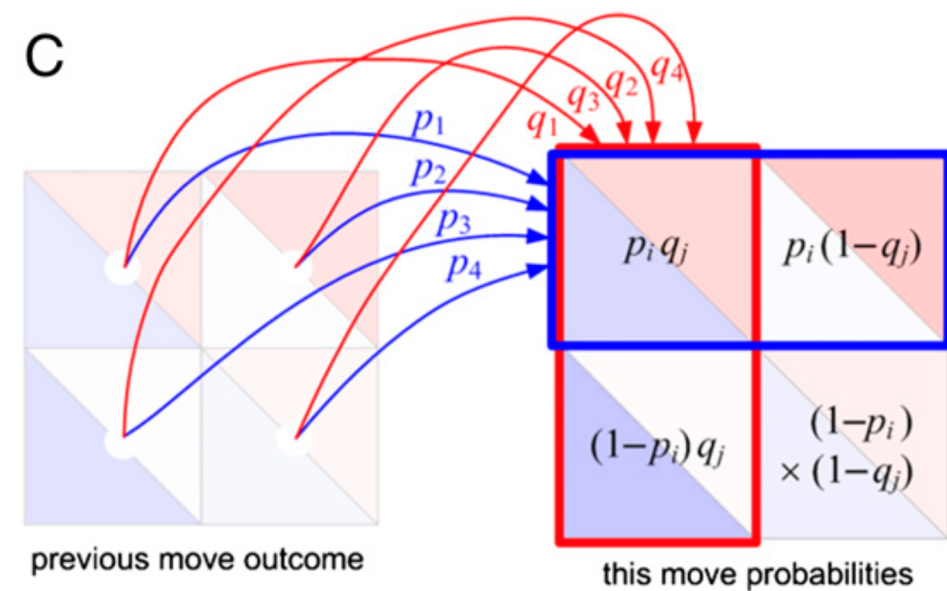
Memory One

- The same game (same allowed moves and same payoff matrices) is indefinitely repeated: Shortest-Memory Player Sets the Rules of the Game



Strategy vector

- $p = (p_1, p_2, p_3, p_4)$ for $xy \in (cc, cd, dc, dd)$
- $q = (q_1, q_2, q_3, q_4)$ for $yx \in (cc, cd, dc, dd)$



Markov transition matrix

$$\begin{bmatrix} p_1 q_1 & p_1(1 - q_1) & (1 - p_1)q_1 & (1 - p_1)(1 - q_1) \\ p_2 q_3 & p_2(1 - q_3) & (1 - p_2)q_3 & (1 - p_2)(1 - q_3) \\ p_3 q_2 & p_3(1 - q_2) & (1 - p_3)q_2 & (1 - p_3)(1 - q_2) \\ p_4 q_4 & p_4(1 - q_4) & (1 - p_4)q_4 & (1 - p_4)(1 - q_4) \end{bmatrix}$$

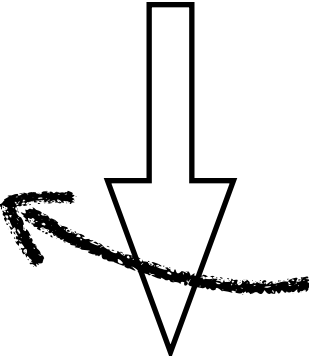
- Markov matrix (all element > 0 , each row adds to 1 \Rightarrow have a (maximum) eigenvalue 1)
- the matrix $M' \equiv M - I$ is singular, with thus zero determinant.
- The stationary vector v of the Markov matrix: $v^T M = v^T$; or $v^T M' = 0$
- $\text{Adj}(M')M' = \det(M')I = 0$

Markov矩阵其余特征值的绝对值小于1的证明

- Markov矩阵M
- 首先，1是M的特征值
- 假设存在M的特征值 $\lambda > 1$ ，则 $Mx = \lambda x$ ，令 x_i 是 x 中最大元素，因为M每一项都大于0，每一行的和为1，即 λx 中的每一项都是 x 的每一个元素的凸组合，所以 λx 中的每一个都不大于 x_i ，而 $\lambda > 1$ ， $\lambda x_i > x_i$ ，与假设矛盾。

Markov transition matrix

$$\begin{bmatrix} p_1 q_1 & p_1(1 - q_1) & (1 - p_1)q_1 & (1 - p_1)(1 - q_1) \\ p_2 q_3 & p_2(1 - q_3) & (1 - p_2)q_3 & (1 - p_2)(1 - q_3) \\ p_3 q_2 & p_3(1 - q_2) & (1 - p_3)q_2 & (1 - p_3)(1 - q_2) \\ p_4 q_4 & p_4(1 - q_4) & (1 - p_4)q_4 & (1 - p_4)(1 - q_4) \end{bmatrix}$$

$$\begin{bmatrix} -1 + p_1 q_1 & p_1(1 - q_1) & (1 - p_1)q_1 & (1 - p_1)(1 - q_1) \\ p_2 q_3 & -1 + p_2(1 - q_3) & (1 - p_2)q_3 & (1 - p_2)(1 - q_3) \\ p_3 q_2 & p_3(1 - q_2) & -1 + (1 - p_3)q_2 & (1 - p_3)(1 - q_2) \\ p_4 q_4 & p_4(1 - q_4) & (1 - p_4)q_4 & -1 + (1 - p_4)(1 - q_4) \end{bmatrix} \begin{matrix} \leftarrow \\ \leftarrow \\ \leftarrow \\ \leftarrow \end{matrix}$$


$$\begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} \\ C_{12} & C_{22} & C_{32} & C_{42} \\ C_{13} & C_{23} & C_{33} & C_{43} \\ C_{14} & C_{24} & C_{34} & C_{44} \end{bmatrix}$$

$$\mathbf{v} \cdot \mathbf{f} \equiv D(\mathbf{p}, \mathbf{q}, \mathbf{f})$$

$$= \det \begin{bmatrix} -1 + p_1 q_1 & -1 + p_1 & -1 + q_1 & f_1 \\ p_2 q_3 & -1 + p_2 & q_3 & f_2 \\ p_3 q_2 & p_3 & -1 + q_2 & f_3 \\ p_4 q_4 & p_4 & q_4 & f_4 \end{bmatrix}$$

$\equiv \tilde{\mathbf{p}}$
 $\equiv \tilde{\mathbf{q}}$

Zero-Determinant Strategies

$$\begin{aligned}
 \mathbf{v} \cdot \mathbf{f} &\equiv D(\mathbf{p}, \mathbf{q}, \mathbf{f}) \\
 &= \det \begin{bmatrix} -1 + p_1 q_1 & -1 + p_1 & -1 + q_1 & f_1 \\ p_2 q_3 & -1 + p_2 & q_3 & f_2 \\ p_3 q_2 & p_3 & -1 + q_2 & f_3 \\ p_4 q_4 & p_4 & q_4 & f_4 \end{bmatrix} \\
 &\quad \equiv \tilde{\mathbf{p}} \quad \equiv \tilde{\mathbf{q}}
 \end{aligned}$$

$$s_X = \frac{\mathbf{v} \cdot \mathbf{S}_X}{\mathbf{v} \cdot \mathbf{1}} = \frac{D(\mathbf{p}, \mathbf{q}, \mathbf{S}_X)}{D(\mathbf{p}, \mathbf{q}, \mathbf{1})}$$

$$s_Y = \frac{\mathbf{v} \cdot \mathbf{S}_Y}{\mathbf{v} \cdot \mathbf{1}} = \frac{D(\mathbf{p}, \mathbf{q}, \mathbf{S}_Y)}{D(\mathbf{p}, \mathbf{q}, \mathbf{1})},$$

$$\mathbf{S}_X = (R, S, T, P)$$

$$\mathbf{S}_Y = (R, T, S, \dot{P})$$

X Unilaterally Sets Y's Score

$$\tilde{\mathbf{p}} = \beta \mathbf{S}_Y + \gamma \mathbf{1}$$

$$p_2 = \frac{p_1(T - P) - (1 + p_4)(T - R)}{R - P}$$

$$p_3 = \frac{(1 - p_1)(P - S) + p_4(R - S)}{R - P}$$

$$s_Y = \frac{(1 - p_1)P + p_4R}{(1 - p_1) + p_4}$$

X Tries to Set Her Own Score

$$\tilde{\mathbf{p}} = \alpha \mathbf{S}_X + \gamma \mathbf{1}$$

$$p_2 = \frac{(1 + p_4)(R - S) - p_1(P - S)}{R - P} \geq 1$$

$$p_3 = \frac{-(1 - p_1)(T - P) - p_4(T - R)}{R - P} \leq 0.$$

$$\mathbf{p} = (1, 1, 0, 0)$$



X Demands and Gets an Extortionate Share

$$\tilde{\mathbf{p}} = \phi[(\mathbf{S}_X - P\mathbf{1}) - \chi(\mathbf{S}_Y - P\mathbf{1})],$$

$$p_1 = 1 - \phi(\chi - 1) \frac{R - P}{P - S}$$

$$(5, 3, 1, 0)$$

$$p_2 = 1 - \phi \left(1 + \chi \frac{T - P}{P - S} \right)$$

$$\mathbf{p} = [1 - 2\phi(\chi - 1), 1 - \phi(4\chi + 1), \phi(\chi + 4), 0]$$

$$p_3 = \phi \left(\chi + \frac{T - P}{P - S} \right)$$

$$s_X = \frac{2 + 13\chi}{2 + 3\chi}, \quad s_Y = \frac{12 + 3\chi}{2 + 3\chi}.$$

$$p_4 = 0$$

$$0 < \phi \leq \frac{(P - S)}{(P - S) + \chi(T - P)}.$$

Special Case

$$\chi = 1, \phi = \frac{1}{5}$$

$$\mathbf{p} = (1, 0, 1, 0)$$

Discussion: X Demands and Gets an Extortionate Share

$$\tilde{p} = \phi[(S_X - P1) - \chi(S_Y - P1)],$$

tends to mutual defection

$$\tilde{p} = \phi[(S_X - R1) - \chi(S_Y - R1)]$$

tends to mutual cooperation

Discussion: A ZD player meets her opponent

X	Y	Result
ZD	A simple evolution player	No dilemma
ZD	An evolution player with a theory in mind	Become an ultimatum game
ZD	ZD	May leads to negotiate

Thanks

