Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent

William H Press and Freeman J Dyson.
Proceedings of the National Academy of Sciences of the United States of America, 2012, 109(26): 10409–10413.

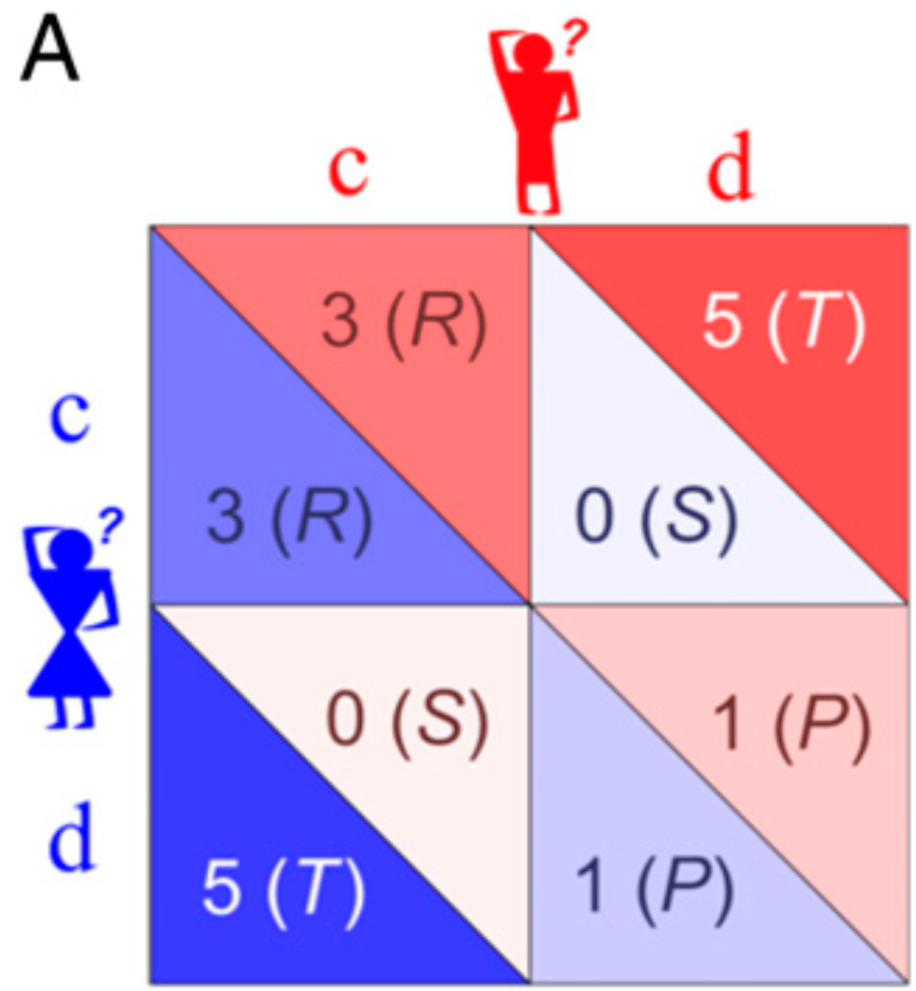
Presented by Wang Jiasheng

Outline

- Preliminaries
- Zero-Determinant Strategies
- Dominate opponent
- Discussion

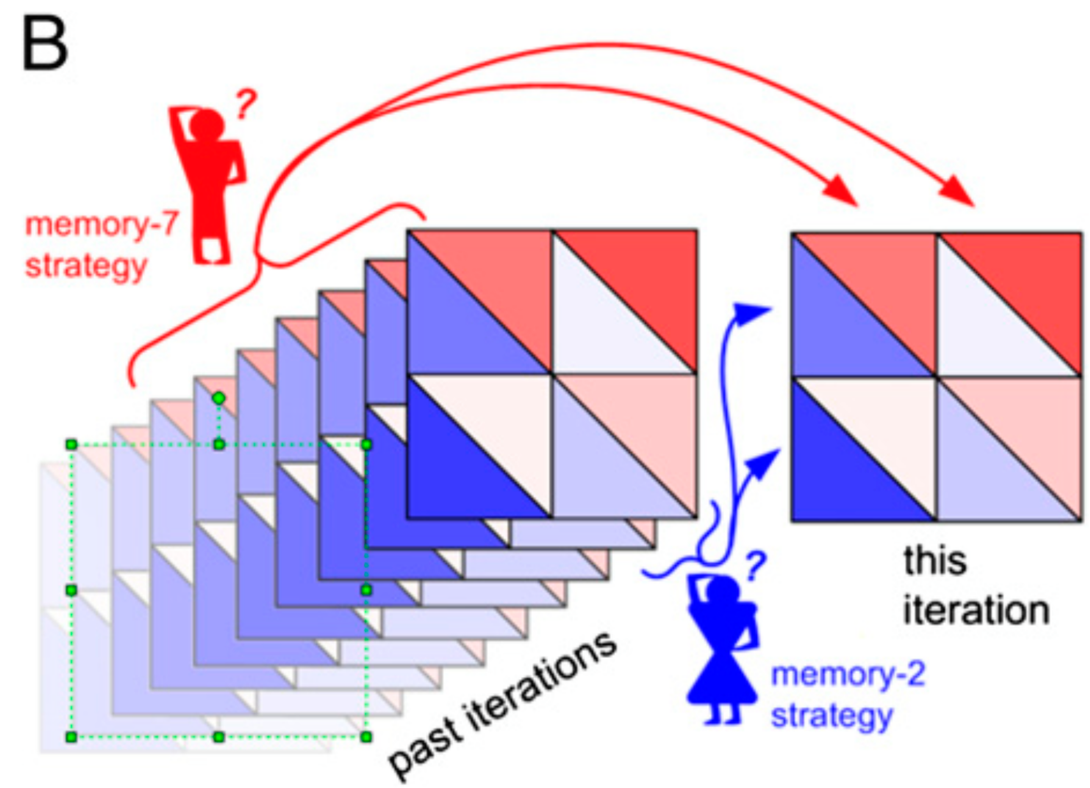
Iterated Prisoner's Dilemma (IPD)

- (T, R, P, S)
- T > R > P > S
- 2R > T + S



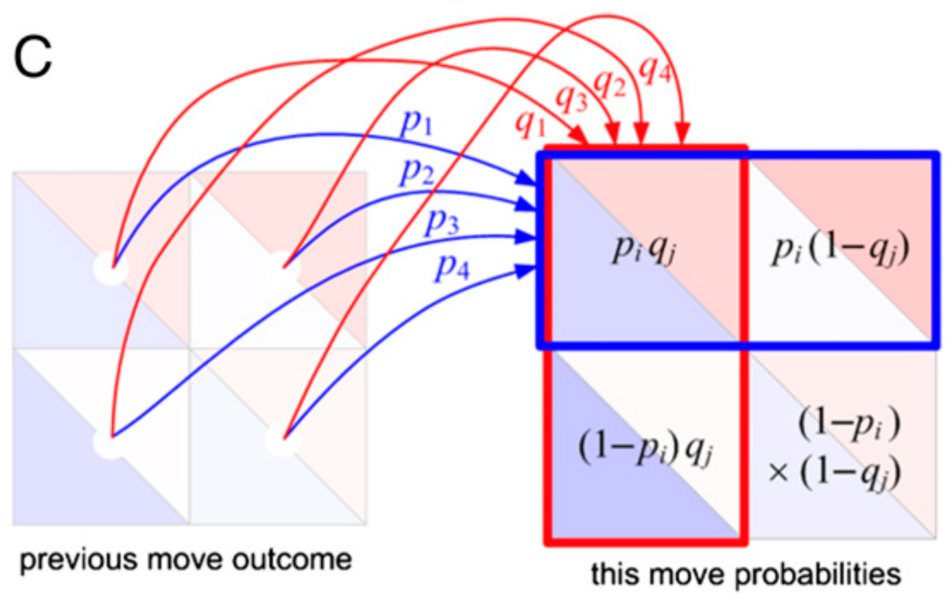
Memory One

- The same game (same allowed moves and same payoff matrices) is indefinitely repeated： Shortest-Memory Player Sets the Rules of the Game



Strategy vector

- p = (p1, p2, p3, p4) for xy∈(cc,cd,dc,dd)
- q = (q1, q2, q3, q4) for yx∈(cc,cd,dc,dd)



Markov transition matrix

$$\begin{bmatrix} p_1q_1 & p_1(1-q_1) & (1-p_1)q_1 & (1-p_1)(1-q_1) \\ p_2q_3 & p_2(1-q_3) & (1-p_2)q_3 & (1-p_2)(1-q_3) \\ p_3q_2 & p_3(1-q_2) & (1-p_3)q_2 & (1-p_3)(1-q_2) \\ p_4q_4 & p_4(1-q_4) & (1-p_4)q_4 & (1-p_4)(1-q_4) \end{bmatrix}$$

- Markov matrix (all element > 0, each row adds to 1 => have a (maximum) eigenvalue 1)
- the matrix $M' \equiv M - I$ is singular, with thus zero determinant.
- The stationary vector v of the Markov matrix： $v^T M = v^T$; or $v^T M' = 0$
- $\mathrm{Adj}(M')M' = \det(M')I = 0$

Markov transition matrix

$$\begin{bmatrix} p_1q_1 & p_1(1-q_1) & (1-p_1)q_1 & (1-p_1)(1-q_1) \\ p_2q_3 & p_2(1-q_3) & (1-p_2)q_3 & (1-p_2)(1-q_3) \\ p_3q_2 & p_3(1-q_2) & (1-p_3)q_2 & (1-p_3)(1-q_2) \\ p_4q_4 & p_4(1-q_4) & (1-p_4)q_4 & (1-p_4)(1-q_4) \end{bmatrix}$$

$$\begin{bmatrix} -1+p_1q_1 & p_1(1-q_1) & (1-p_1)q_1 & (1-p_1)(1-q_1) \\ p_2q_3 & -1+p_2(1-q_3) & (1-p_2)q_3 & (1-p_2)(1-q_3) \\ p_3q_2 & p_3(1-q_2) & -1+(1-p_3)q_2 & (1-p_3)(1-q_2) \\ p_4q_4 & p_4(1-q_4) & (1-p_4)q_4 & -1+(1-p_4)(1-q_4) \end{bmatrix}$$

$$\begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} \\ C_{12} & C_{22} & C_{32} & C_{42} \\ C_{13} & C_{23} & C_{33} & C_{43} \\ C_{14} & C_{24} & C_{34} & C_{44} \end{bmatrix}$$

$$\mathbf{v}\cdot\mathbf{f} \equiv D(\mathbf{p},\mathbf{q},\mathbf{f})$$

$$= \det \begin{bmatrix} -1+p_1q_1 & -1+p_1 & -1+q_1 & f_1 \\ p_2q_3 & -1+p_2 & q_3 & f_2 \\ p_3q_2 & p_3 & -1+q_2 & f_3 \\ p_4q_4 & p_4 & q_4 & f_4 \end{bmatrix}$$

$$\equiv \tilde{\mathbf{p}} \qquad \equiv \tilde{\mathbf{q}}$$

Zero-Determinant Strategies

$$\mathbf{v}\cdot\mathbf{f} \equiv D(\mathbf{p},\mathbf{q},\mathbf{f})$$

$$= \det\begin{bmatrix} -1+p_1q_1 & -1+p_1 & -1+q_1 & f_1 \\ p_2q_3 & -1+p_2 & q_3 & f_2 \\ p_3q_2 & p_3 & -1+q_2 & f_3 \\ p_4q_4 & p_4 & q_4 & f_4 \end{bmatrix}$$

$$\underbrace{}_{\equiv\,\tilde{\mathbf{p}}} \qquad \underbrace{}_{\equiv\,\tilde{\mathbf{q}}}$$

$$s_X = \frac{\mathbf{v}\cdot\mathbf{S}_X}{\mathbf{v}\cdot\mathbf{1}} = \frac{D(\mathbf{p},\mathbf{q},\mathbf{S}_X)}{D(\mathbf{p},\mathbf{q},\mathbf{1})}$$

$$s_Y = \frac{\mathbf{v}\cdot\mathbf{S}_Y}{\mathbf{v}\cdot\mathbf{1}} = \frac{D(\mathbf{p},\mathbf{q},\mathbf{S}_Y)}{D(\mathbf{p},\mathbf{q},\mathbf{1})},$$

$$\mathbf{S}_X = (R,S,T,P)$$

$$\mathbf{S}_Y = (R,T,S,P)$$

X Unilaterally Sets Y's Score

$$\tilde{\mathbf{p}} = \beta \mathbf{S}_Y + \gamma \mathbf{1}$$

$$p_2 = \frac{p_1(T-P) - (1+p_4)(T-R)}{R-P}$$

$$p_3 = \frac{(1-p_1)(P-S) + p_4(R-S)}{R-P}.$$

$$s_Y = \frac{(1-p_1)P + p_4 R}{(1-p_1) + p_4}.$$

X Tries to Set Her Own Score

$$\tilde{\mathbf{p}} = \alpha \mathbf{S}_X + \gamma \mathbf{1}$$

$$p_2 = \frac{(1+p_4)(R-S) - p_1(P-S)}{R-P} \geq 1$$

$$p_3 = \frac{-(1-p_1)(T-P) - p_4(T-R)}{R-P} \leq 0.$$

$$\mathbf{p} = (1,1,0,0)$$

X Demands and Gets an Extortionate Share

$$\tilde{\mathbf{p}} = \phi[(\mathbf{S}_X - P\mathbf{1}) - \chi(\mathbf{S}_Y - P\mathbf{1})],$$

$$p_1 = 1 - \phi(\chi - 1)\frac{R-P}{P-S}$$

$$p_2 = 1 - \phi\left(1 + \chi\frac{T-P}{P-S}\right)$$

$$p_3 = \phi\left(\chi + \frac{T-P}{P-S}\right)$$

$$p_4 = 0$$

$$0 < \phi \le \frac{(P-S)}{(P-S) + \chi(T-P)}.$$

$$(5, 3, 1, 0)$$

$$\mathbf{p} = [1 - 2\phi(\chi - 1), 1 - \phi(4\chi + 1), \phi(\chi + 4), 0]$$

$$s_X = \frac{2 + 13\chi}{2 + 3\chi}, \quad s_Y = \frac{12 + 3\chi}{2 + 3\chi}.$$

Special Case

$$\chi = 1, \phi = \frac{1}{5}$$

$$p = (1, 0, 1, 0)$$

Discussion: X Demands and Gets an Extortionate Share

$$\tilde{\mathbf{p}} = \phi[(\mathbf{S}_X - P\mathbf{1}) - \chi(\mathbf{S}_Y - P\mathbf{1})],$$

tends to mutual defection

$$\tilde{p} = \phi[(S_X - R1) - \chi(S_Y - R1)]$$

tends to mutual cooperation

Discussion: A ZD player meets her opponent

X	Y	Result
ZD	A simple evolution player	No dilemma
ZD	An evolution player with a theory in mind	Become an ultimatum game
ZD	ZD	May leads to negotiate

Thanks